STATIONARY THEORY OF A VISCOSITY EXPLOSION IN NON-NEWTONIAN MEDIA

V. I. Naidenov

UDC 532.135

A theory is proposed for a viscosity thermal explosion, a progressing growth of fluid viscosity under stationary external conditions.

The hydrodynamic effects of thermal nature, due to the dependence of the dissipative heat liberation function on the temperature, were detected in [1-3]. The crux of these effects is that no equality holds between the quantity of heat being liberated in a fluid because of viscous friction and the elimination of this heat through the wall by molecular heat conduction. The heat-balance equations in the cases under consideration are analogous to the equations of the stationary theory of a thermal explosion [4], in which connection many results of this theory have been carried over to the flows of viscous media. The nonlinear temperature dependence of convective heat transfer can also result [5-7] in hydrodynamic effects associated with losses of thermal stability, and particularly, in a viscosity explosion. A viscosity explosion of thermal nature (in contrast to the analogous phenomenon caused by the dependence of the viscosity on the degree of transformation [8]) was first detected in investigation of viscous fluid flow in a circular pipe whose walls are cooled by a stationary heat flux [6]. It should be noted that the theory of this phenomenon was based on quite simplified equations of motion and heat transmission which are not correct for substantial heat fluxes through the pipe walls; consequently there is a need to construct a refined theory of this effect that would be of applied value in polymer reprocessing associated with their cooling (rolling, casting under pressure, etc.). Moreover, the mechanism of thermal instability formation that is characteristic for the phenomenon detected can be relied upon for an explanation of the irregular flow of polymer masses observed during their extrusion.

Let us consider the flow of a non-Newtonian fluid in a plane channel of width 2h whose walls are either maintained at a temperature linearly dependent on the longitudinal coordinate x, $T_w = T_0 - Ax$, or a constant heat removal $\lambda \partial T/\partial y = -q_w$ is accomplished from the channel wall. We assume $A > 0$, $q_w > 0$, i.e., fluid cooling occurs. We take the rheological equation of state in the form of a power dependence of the friction stress on the velocity gradient, and we approximate the effective viscosity by the Reynolds relationship

$$\tau(x, y) = k_0 \left| \frac{du}{dy} \right|^{n-1} e^{-\beta(T-T_0)} \frac{du}{dy}. \quad (1)$$

The temperature field is two-dimensional in the case under consideration, i.e., the heat flux vector $\vec{q} = \left\{ -\lambda \frac{\partial T}{\partial x}, -\lambda \frac{\partial T}{\partial y} \right\}$ has both components different from zero, which distinguishes this problem from other analogous problems. Moreover, the convective terms in the heat transfer equation are not zero identically and are fundamental for the investigation of the thermal instability mechanism. As is shown in [9], the problem formulated is self-similar if we seek the class of nonisothermal flows satisfying the condition q_x = const. In this case it turns out that a motion regime of an incompressible medium exists for which the velocity field is one-dimensional, i.e., $u_x = u(y)$, $v_y \equiv 0$, despite the fact that the pressure, temperature, and viscosity fields depend on both coordinates.

We therefore consider the system of equations

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}, \quad \frac{\partial p}{\partial y} = \frac{\partial \tau}{\partial x}, \quad \lambda \Delta T = \rho c_p \operatorname{grad} T. \quad (2)$$

Central Scientific-Research and Planning and Design Institute of Mechanization and Energetics of the Wood Industry. Khimki. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 46, No. 5, pp. 837-845, May, 1984. Original article submitted March 21, 1983.

 0022-0841/84/4605-0612$08.50

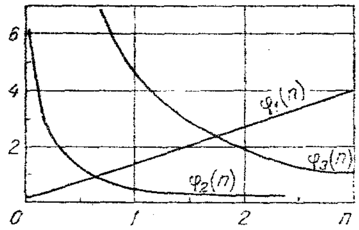


Fig. 1. Dependence of the critical conditions for a viscosity explosion on the flow index.

According to the above, we take the boundary conditions for the temperature in the form

$$\lambda \frac{\partial T}{\partial y}\bigg|_{y=\pm h} = -q_w,\ T|_{y=\pm h} = T_w = T_0 - Ax. \tag{3}$$

Utilizing the hypothesis of a power-law fluid adhering to the channel wall, we will have $u|_{y=\pm h}=0$ for the velocity. In the case of assigning a heat flux at the channel walls, we assume that the mean bulk fluid temperature at the entrance to the heat-exchange section is known

$$T_0 = \frac{1}{2hu_0}\int_{-h}^{h} T(0,\ y)\,u(y)\,dy,\ u_0 = \frac{1}{2h}\int_{-h}^{h} u(y)\,dy. \tag{4}$$

We consider a domain of stabilized heat exchange, i.e., $K = L/h \gg 1$. We will seek the temperature distribution in the form $T(x, y) = T_0 - Ax + T_1(y)$, where $T_1(y)$ is an unknown function while the friction stress in the flow is

$$\tau(x,\ y) = z(y)\,e^{\beta Ax},\ z(y) = k_0\left|\frac{du}{dy}\right|^{n-1} e^{-\beta T_1}\,\frac{du}{dy},$$

and we obtain for the determination of z(y) from the equation of motion

$$z'' = \beta^2 A^2 z,\ z(y) = -D_1(\beta A)^{-1}\,\mathrm{sh}\,\beta Ay + D_2\,\mathrm{ch}\,\beta Ay,$$

where D_1, D_2 are constants of integration.

From the condition of flow symmetry there follows $D_2 = 0$. Therefore, the friction stress in a nonisothermal flow of a power-law fluid is expressed by

$$\tau(x,\ y) = -D_1(\beta A)^{-1}\,\mathrm{sh}\,\beta Ay\,e^{\beta Ax}.$$

Correspondingly, the pressure gradient equals

$$\frac{\partial p}{\partial x} = -D_1\,\mathrm{ch}\,\beta Ay\,e^{\beta Ax},\ \frac{\partial p}{\partial y} = -D_1\,\mathrm{sh}\,\beta Ay\,e^{\beta Ax}.$$

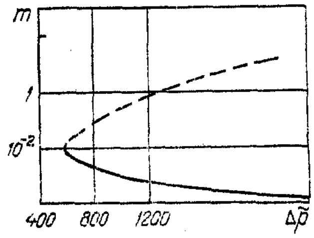
The ratio between the transverse and longitudinal pressure drops has the form

$$\frac{\partial p}{\partial y}\bigg/\frac{\partial p}{\partial x} = \mathrm{th}\,\beta Ay.$$

Since the flow of a power-law fluid is considered in channels of small width, then $\beta Ah \ll 1$ and in many cases $\frac{\partial p}{\partial y}\big/\frac{\partial p}{\partial x} \ll 1$. The ratio between the friction stress in the flow and the friction stress on the walls is $\tau/\tau_w = \mathrm{sh}\,\beta Ay/\mathrm{sh}\,\beta Ah$. As $\beta \to 0$ (flow with a constant coefficient of viscosity), $\tau/\tau_w \to y/h$.

We shall consider a given mean pressure drop over the cross section on a section of length L:

$$\Delta p = -\frac{p(L) - p(0)}{L} = -\frac{1}{Lh}\int_0^h\int_0^L \frac{\partial p}{\partial x}(x,\ y)\,dx\,dy. \tag{5}$$



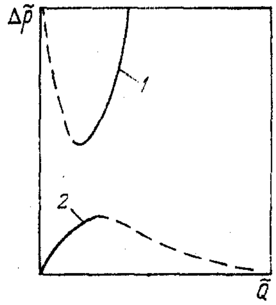


Fig. 2 Fig. 3

Fig. 2. Dependence of the longitudinal temperature gradient on the pressure drop.

Fig. 3. Dependence of the pressure drop on the fluid mass flow rate.

From the relationship (5) we determine the constant of integration D_1:

$$D_1 = \Delta p \frac{m^2 k}{\operatorname{sh} m(\exp(mk) - 1)}, \quad m = \beta A h.$$

As $\beta \to 0$, $D_1 \to \Delta p$ we obtain the pressure drop in the case of a plane isothermal power-law fluid flow.

Taking account of the expression for the fluid temperature and the friction stress in the flow, we find from (2)

$$k_0 \left| \frac{du}{dy} \right|^{n-1} \frac{du}{dy} e^{-\beta T_1} = -\frac{D_1 \operatorname{sh} \beta A y}{\beta A},$$

$$\lambda \frac{d^2 T_1}{dy^2} = -\rho c_p u A. \tag{6}$$

From the physical sense it is clear that $du/dy \leqslant 0$, and therefore

$$-\frac{du}{dy} = \sqrt[n]{\frac{D_1 \operatorname{sh} \beta A y}{k_0 \beta A}}\, e^{\frac{\beta T_1}{n}}$$

or

$$\lambda \frac{d^3 T_1}{dy^3} = \rho c_p A \sqrt[n]{\frac{D_1 \operatorname{sh} \beta A y}{k_0 \beta A}}\, e^{\frac{\beta T_1}{n}}.$$

Integrating the second equation of system (6) across the channel width, we obtain

$$q_w = \rho c_p A \int_0^h u(y)\, dy = \rho c_p A h u_0. \tag{7}$$

Integrating (7) by parts, we determine

$$q_w = -\rho c_p A \int_0^h y \frac{du}{dy} dy = \rho c_p A \int_0^h y \sqrt[n]{\frac{D_1 \operatorname{sh} \beta A y}{k_0 \beta A}}\, e^{\frac{\beta T_1}{n}} dy.$$

Let us introduce the dimensionless variables

$$\xi = y/h, \ \theta = \beta T_1, \ \tilde{q}_w = \frac{\beta q_w h}{\lambda}, \ \Delta\tilde{p} = \frac{\rho c_p h}{\lambda} \sqrt[n]{\frac{\Delta p h}{k_0}}.$$

Going over to these new variables, we obtain the system of equations

$$\theta''' = \tilde{q}_w \Psi(\theta) \sqrt[n]{\operatorname{sh} m\xi}\, e^{\theta/n},$$

$$\Delta\tilde{p} = \tilde{q}_w \Psi(\theta)\, m^{\frac{1-n}{n}} \sqrt[n]{\frac{\operatorname{sh} m(\exp mk - 1)}{m^2 k}},$$

$$\Psi(\theta) = \Big[\int_0^1 \xi \sqrt[n]{\operatorname{sh} m\xi}\, e^{\theta/n} d\xi\Big]^{-1}. \tag{8}$$

Let us formulate boundary conditions for the system (8).

Since the fluid velocity is zero on the channel walls, then $\theta''(1) = 0$ follows from the equations of the system (6). The flow symmetry condition yields $\theta'(0) = 0$. In the case of assigning a heat flux on the wall, there follows from (4):

$$\int_0^1 \theta\theta'' d\xi = \theta(1)\,\theta'(1) - \int_0^1 (\theta')^2 d\xi = 0.$$

Since $\theta'(1) = -\tilde{q}_w$, then $\theta(1) = -\frac{1}{\tilde{q}_w}\int_0^1 (\theta')^2 d\xi$. In the case of assigning a fluid temperature T_w on the wall it is natural to take $T_1|_{y=\pm h} = 0$ or $\theta(1) = 0$.

Thus, we will consider (8) with two kinds of boundary conditions:

$$\text{a) } \theta'(0) = 0,\ \theta''(1) = 0\ \ \theta(1) = -\frac{1}{\tilde{q}_w}\int_0^1 (\theta')^2 d\xi,$$

$$\text{b) } \theta'(0) = 0,\ \theta''(1) = 0,\ \theta(1) = 0.$$

Let us consider the integrodifferential equations (8) in combination with the boundary-value problem a). In this case the given parameters are $\Delta\tilde{p}$, $\tilde{q}_w$, n, k. The quantities θ, m are subject to determination. After finding the dimensionless excess temperature θ and the temperature gradient m along the flow, the fluid velocity is evaluated from the formula

$$w(\xi) = -m^{-1}\theta'',\ \ w(\xi) = \rho c_p h \lambda^{-1} u.$$

There is a dependence $\tilde{q}_w = m\mathrm{Pe}$ between the introduced dimensionless parameters m, $\tilde{q}_w$ and the Peclet number $\mathrm{Pe} = u_0 h \rho c_p \lambda^{-1}$. Let us show that there exists a critical value of the dimensionless pressure drop $\Delta\tilde{p}^*$, such that for $\Delta\tilde{p} < \Delta\tilde{p}^*$ there is no solution of the equations. We assume that a solution exists for any positive values of $\Delta\tilde{p}$. We make the change of variable

$$\theta^*(\xi) = \theta(\xi) - \theta(1).$$

The first equation of system (8) becomes

$$\theta^{*\prime\prime\prime} = \tilde{q}_w \Psi(\theta^*) \sqrt[n]{\operatorname{sh} m\xi}\, e^{\theta^*/n} \tag{9}$$

with the obvious boundary conditions $\theta^{*\prime}(0) = 0$, $\theta^*(1) = 0$, $\theta^{*\prime\prime}(1) = 0$. We construct the Green's function of the operator $\theta^{*\prime\prime\prime} = 0$ with these boundary conditions

$$G(\xi, t) = \begin{cases} t - 0.5(t^2 + \xi^2), & \xi \leqslant t, \\ t(1 - \xi), & \xi \geqslant t \end{cases}$$

and we write the solution of (9) in the form of the integral equation

$$\theta^*(\xi) = \tilde{q}_w \Psi(\theta^*) \int_0^1 G(\xi, t) \sqrt[n]{\operatorname{sh} mt}\, e^{\theta^*(t)/n} dt.$$

From the properties of the Green's function there follows $G \geqslant 0$, $\partial G/\partial\xi \leqslant 0$, $\partial^2 G/\partial\xi^2 \leqslant 0$. The solutions of (9) will possess an analogous property, i.e., $\theta^* \geqslant 0$, $\theta^{*\prime} \leqslant 0$, $\theta^{*\prime\prime} \leqslant 0$. Since the function $\theta^*(\xi)$ turns out to be concave on the segment [0, 1], then the inequality $\theta^*(\xi) \geqslant \theta^*(0)(1 - \xi)$ is valid. To evaluate $\theta^*(0)$ we have the equation

$$\theta^*(0) = \tilde{q}_w - 0.5\tilde{q}_w \Psi(\theta^*) \int_0^1 t^2 \sqrt[n]{\operatorname{sh} mt}\, e^{\theta^*/n} dt,$$

from which the inequality $\theta^*(0) \geqslant 0.5\tilde{q}_w$, $\theta^*(\xi) \geqslant 0.5\tilde{q}_w(1 - \xi)$ follows. Furthermore, we consider the Cauchy problem for the equation $\varphi''' = 0$, $\varphi(1) = 0$, $\varphi'(1) = -\tilde{q}_w$, $\varphi''(1) = 0$. The solution of this problem is evident: $\varphi = \tilde{q}_w(1-\xi)$. Since $\theta^{*'''} \geqslant 0$, then $\omega''' \geqslant 0$, where $\omega = \theta^* - \varphi$. Integrating the differential inequality $\omega''' \geqslant 0$ successively on the segment [0, 1] with the boundary conditions taken into account for θ^*, φ, we obtain $\omega \leqslant 0$, $\theta^*(\xi) \leqslant \tilde{q}_w(1-\xi)$. Therefore, for any values of the parameters $\Delta\tilde{p}$, $\tilde{q}_w$, n, k under the assumption of the existence of a solution of (9), we have the inequality $0.5\tilde{q}_w(1-\xi) \leqslant \theta^*(\xi) \leqslant \tilde{q}_w(1-\xi)$. After determining $\theta^*(\xi)$ the desired solution $\theta(\xi)$ is evaluated from the formula

$$\theta(\xi) = \theta^*(\xi) - \frac{1}{\tilde{q}_w}\int_0^1 (\theta^{*\prime})^2 d\xi.$$

Let us consider the second equation of system (8)

$$\Delta\tilde{p} = \tilde{q}_w m^{\frac{1-n}{n}} \Psi(\theta^*) \sqrt[n]{\frac{\operatorname{sh} m(\exp(mk)-1)}{m^2 k}} \; e^{\frac{1}{\tilde{q}_w n}\int_0^1 (\theta^{*\prime})^2 dt}$$

from which the inequality

$$\Delta\tilde{p} \geqslant \tilde{q}_w m^{\frac{1-n}{n}} \Psi[\tilde{q}_w(1-t)] \sqrt[n]{\frac{\operatorname{sh} m(\exp(mk)-1)}{m^2 k}} \geqslant \tilde{q}_w e^{-\tilde{q}_w/n} f(m,\ k),$$

$$f(m,\ k) = m^{\frac{1-n}{n}} \sqrt[n]{\frac{\operatorname{sh} m(\exp(mk)-1)}{m^2 k}} \left(\int_0^1 t \sqrt[n]{\operatorname{sh} mt}\, dt\right)^{-1}$$

follows. We study the properties of function f(m, k) constructed. As m changes in the interval $(0, \infty)$ $\lim_{m\to 0} f = \infty$, $\lim_{m\to\infty} f = \infty$. Since function f(m, k) is continuous, then f(m, k) has a positive minimum in the interval under consideration. Let m* denote the point in the interval (0, ∞) at which f(m) has a minimum, i.e., f'(m*) = 0. We seek the solution of the transcendental equations f'(m*) = 0 in the form $m^* = a^* k^{-1}$ (because of the awkwardness of this equation we do not write it down explicitly). Substituting this value into the equation f'(m*) = 0 and retaining terms of lowest order of smallness for the determination of a^*, we obtain the equation

$$\frac{1-e^{-a^*}}{a^*} = \frac{1}{n+1}. \qquad (10)$$

The solution of the transcendental equation as a function of the flow index n will be denoted by $a^* = \varphi_1(n)$. The value of the function f(m*) for $k \gg 1$ is given by the expression

$$f(a^*) = k\varphi_3(n), \quad \varphi_3(n) = \varphi_2(n) \sqrt[n]{\frac{1}{n+1-\varphi_1(n)}} \frac{2n+1}{n} k,$$

$$\varphi_2(n) = [\varphi_1(n)]^{-1}.$$

The dependences $\varphi_1(n)$, $\varphi_2(n)$, $\varphi_3(n)$ are represented in Fig. 1.

It follows from the above that for $\Delta\tilde{p} < \tilde{q}_w e^{-\tilde{q}_w/n}$ the min f(m, k) of the solution of the system (8) does not exist. Therefore, there is a $\Delta\tilde{p}^*$, a critical value of the pressure drop for which there are no real solutions of the system of integrodifferential equations (8). Since $\theta^* \sim \tilde{q}_w$, then as $\tilde{q}_w \to 0$ we obtain asymptotic expressions for $\Delta\tilde{p}^*$, m^*, Pe*:

$$m^* = \varphi_1(n) k^{-1}, \ \Delta\tilde{p}^* = k\varphi_3(n)\tilde{q}_w, \ \mathrm{Pe}^* = k\varphi_2(n)\tilde{q}_w. \qquad (11)$$

For subcritical values of the pressure drop $\Delta\tilde{p} > \Delta\tilde{p}^*$ and $\tilde{q}_w \to 0$ the relation between the dimensionless temperature gradient along the channel and the pressure drop is given by the expresssion $\Delta\tilde{p} = \tilde{q}_w f(m,\ k)$, from which there follows that for a given $\Delta\tilde{p}$ two values exist for m, and therefore, also for the other flow characteristics (the heat elimination coefficient, fluid temperature, mass flow rate, etc.).

In other words, ambiguity of the solutions of the system of equations (8) holds for subcritical values of the pressure drop. Analysis of viscous flow stability shows that a solution with $d\Delta p/du_0 < 0$ (the pressure drop diminishes as the fluid mass flow rate increases) turns out to be unstable with respect to thermal perturbations (dashed curves in Figs. 2 and 3) and stable regimes with $d\Delta p/du_0 > 0$ are physically realizable (solid curves). For the problem a) these regimes correspond to the equilibrium state with a high fluid temperature.

Let us turn to a physical analysis of the results obtained. The heat transmission mechanism during power-law fluid motion in a channel, described by (8), comprises the following. Heat is supplied to the heat-exchanger section by forced convection and is eliminated to the channel walls by molecular heat conduction. The quantity of heat released by the fluid is proportional to the product of the flow velocity by the temperature gradient along the channel. Under stationary conditions this quantity of heat equals the heat elimination intensity from the channel walls. As the pressure drop diminishes, and therefore, the mass flow rate of the fluid also, the temperature gradient increases and the fluid releases that quantity of heat that is needed to maintain the thermal equilibrium. For pressure drops less than the critical, the drop in the liquid mass flow rate occurs more rapidly than the growth of the temperature gradient, the thermal equilibrium is spoiled, and the regime with progressive increase in the viscosity with time sets in. This physical phenomenon is defined as a viscosity explosion. It follows from the above that this phenomenon also holds in non-Newtonian fluids, and in all visibility, in all running media with a sufficiently strong dependence of the effective viscosity on the temperature. On the basis of the properties of parabolic equations [10], it can be assumed that large viscosity gradients can be achieved in comparatively short time segments, i.e., this phenomenon has a finite period of induction and is actually realized in the form of an explosion. Precisely the impossibility of realizing thermal equilibrium for critical values of the parameters and a finite period of induction make the analogy with the phenomenon of a thermal explosion legitimate.

From the meaning of the results obtained it is evident that the phenomenon of a viscosity explosion will not hold for a constant coefficient of viscosity.

Let us note the influence of being non-Newtonian on the characteristics of the viscosity explosion. As the flow index n increases the critical value of the pressure drop diminishes, correspondingly, the critical value of the Peclet criterion also diminishes, where $\Delta\tilde{p}^* = \frac{\varphi_3(n)}{\varphi_1(n)}\mathrm{Pe}^*$. The value of the dimensionless temperature gradient along the flow m* increases almost according to a rectilinear law (the curve $\varphi_1(n)$) as n increases. For pseudoplastic fluids (n < 1) the critical value Pe* increases abruptly as n diminishes while the critical pressure drop increases rather more weakly.

The critical values of the pressure drop and Peclet number increase with the growth of $\tilde{q}_w$ for small heat loads. Let us present a numerical example. We assume that in the heat-exchange section of length L = 0.5 m and width 2h = 10^{-2} m there is a polyethylene melt 10802-020 [11] with temperature T_0 = 160°C at the entrance. Constant heat release of intensity q_w = 0.838 kW/m^2 is accomplished from the channel walls. The thermophysical properties of the polyethylene in the temperature range under consideration are the following: c_p = 2.22 kJ/kg·deg, $\lambda = 18.85 \cdot 10^{-5}$ kW/m·deg, ρ = kg/m^3, $k_0 = 29.43 \cdot 10^2$ N·sec/m^2, β = 0.05 1/deg.

Since (11) are obtained under the assumption of small heat loads $\tilde{q}_w \ll 1$, we then assume that the flow velocity gradients are small at the time of the viscosity explosion, and therefore, the melt manifests the properties of a Newtonian fluid (n = 1). It is subsequently easy to confirm this assumption. For n = 1 the formulas (11) will have the following dimensional form

$$p^*(0) - p^*(L) = \frac{4.635\beta q_w k_0 L^2}{\rho c_p h^3},$$

$$u_0^* = 0.627\frac{\beta q_w L}{\rho c_p h}.$$

Substituting the initial data we obtain $p^*(0) - p^*(L)$ = 0.64 MPa, $u_0^* = 15 \cdot 10^{-4}$ m/sec. The velocity gradient is u*/h = 0.3 sec, and in this range of values the melt will actually behave as a Newtonian fluid.

At the exit from the channel T_1 = 130°C at the time of the viscosity explosion. Let us note the following circumstance. Taking account of the inequality $0.5q_w(1-\xi) \leqslant \theta^* \leqslant \tilde{q}_w(1-\xi)$, it is easy to obtain the estimate $\Delta\tilde{p}^* \sim \exp - \tilde{q}_w$ for large values of $\tilde{q}_w$ from the second equation of the system, i.e., the critical value of the pressure drop does not depend monotonically on the magnitude of the heat flux and drops for sufficiently large values of this parameter. This fact is not accidental and underlies the other physical phenomenon, the phenomenon of the convective thermal explosion.

Let us consider the boundary-value problem (8) with the conditions b). In this case $\theta(1)=0,\ \theta^*(\xi)=\theta(\xi)$.

For the pressure drop we will have the following estimate

$$\Delta\tilde{p}\leqslant\tilde{q}_w m^{\frac{1-n}{n}}\sqrt[n]{\frac{\operatorname{sh} m\,(\exp(mk)-1)}{m^2k}}\left[\int_0^1 t\sqrt[n]{\operatorname{sh} mt}\,e^{0,5\tilde{q}_w(1-t)/n}dt\right]^{-1}=F(\tilde{q}_w).$$

We recall that the wall temperature T_w is given in the problem under consideration, and therefore m is a known quantity, while $\tilde{q}_w$ is to be determined. For the changes $0<\tilde{q}_w<\infty$ the function $F(\tilde{q}_w)$ has a positive maximum, and hence there is a $\Delta\tilde{p}^{**}$ above which there is no solution of the system (8) with the boundary conditions b). Physically, the absence of solutions of (8) means that the convective heat transfer along the axis of a plane channel grows more rapidly for post-critical pressure drops than the release of this heat to the walls by molecular heat conduction. It is natural to define this phenomenon as a convective thermal explosion in contrast to the analogous phenomenon due to the temperature dependence of the heat dissipation [2]. In this case the quantity of heat being released by the moving fluid because of convection is proportional to the channel volume, and the heat elimination by heat conduction is proportional to the outer surface; hence, as in thermal explosion problems there is a critical channel diameter above which disturbance of the thermal stability occurs, where the variable fluidity of the medium plays the part of a reaction constant. Ambiguity holds for the subcritical flow regimes $(\Delta\tilde{p}<\tilde{p}^{**})$. It can be shown that the high-temperature regimes for which $d\Delta p/du_0<0$ are unstable. Therefore, a nonmonotonic estimate of the dependence of the critical pressure drop $\Delta\tilde{p}^*$ on the magnitude of the heat flux in the viscous explosion phenomenon is explained by the superposition of the two hydrodynamic effects detected. The subscritical flow regimes for the problems a) and b) (solid curves 1 and 2, respectively) are shown in Fig. 3.

NOTATION

x, y, coordinates; u, T, fluid velocity and temperature; p, τ, pressure and friction stress, respectively, in the fluid flow; λ, heat-conduction coefficient; ρ, c_p, density and specific heat; k_0, n, rheological constants; L, h, channel length and width, respectively; and β, a parameter proportional to the activation energy of the viscous flow.

LITERATURE CITED

1. S. A. Kaganov, "On steady laminar incompressible fluid flow in a plane channel and circular cylindrical tube taking into account the heat of friction and the temperature dependence of the viscosity," Zh. Prikl. Mekh. Tekh. Fiz., No. 3, 96-99 (1962).
2. S. A. Bostandzhiyan, A. G. Merzhanov, and S. I. Khudyaev, "A hydrodynamic thermal explosion," Dokl. Akad. Nauk SSSR, 163, No. 1, 133-136 (1965).
3. A. G. Merzhanov, A. P. Posetsel'skii, A. M. Stolin, and A. S. Shteinberg, "Experimental realization of a hydrodynamic thermal explosion," Dokl. Akad. Nauk SSSR, 210, No. 1, 52-54 (1974).
4. D. A. Frank-Kamenetskii, Diffusion and Heat Transfer in Chemical Kinetics, Plenum Publ. (1969).
5. N. B. Aleksapol'skii and V. I. Naidenov, "Critical phenomena for nonisothermal viscous fluid flows in tubes," Teplofiz. Vys. Temp., 17, No. 4, 783-790 (1979).
6. N. B. Aleksapol'skii and V. I. Naidenov, "Viscosity explosion for nonisothermal incompressible fluid motion," Zh. Prikl. Mekh. Tekh. Fiz., No. 1, 94-97 (1980).
7. Y. T. Shan and I. R. A. Pearson, "Stability of nonisothermal flow in channels," Chem. Eng. Sci., 29, No. 6, 1485-1493 (1974).
8. D. A. Vaganov, "Critical phenomena caused by the change in viscosity with the degree of transformation," Zh. Prikl. Mekh. Tekh. Fiz., No. 2, 168-171 (1975).
9. V. I. Naidenov, "On self-similarity of a convective heat-transfer problem," Zh. Prikl. Mekh. Tekh. Fiz., No. 5, 152-153 (1974).
10. A. I. Vol'pert and S. I. Khudyaev, Analysis in Classes of Discontinuous Functions and Equations of Mathematical Physics [in Russian], Nauka, Moscow (1975), pp. 245-273.
11. Thermophysical and Rheological Characteristics of Polymers (Handbook) [in Russian], Naukova Dumka, Kiev (1977), pp. 79-100.